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A COMPARISON OF GEOGRAPHIC INTERVENTION GROUPING METHODS FOR INFRASTRUCTURE INTERVENTION PLANNING ACROSS MULTIPLE NETWORKS

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Abstract: Interventions on infrastructure networks in municipalities cause disruptions to the service provided by the network that requires the intervention. They also cause disruptions to the service provided by other networks that have to be at least partially shut down so that the intervention can be executed. Due to these effects, there is substantial benefit to be obtained by grouping interventions on all networks that are spatially close to one another, i.e. work programs for spatially close networks should be developed together. This benefit is principally due to reduced interruption to services and reduced costs of intervention. The challenge of determining such combined optimal work programs is made more difficult as it requires quantification of the value of lost services, which depends on how different stakeholders value the services as well as how the services are interrupted. In this paper the difference between two methodologies to be used to develop work programs on spatially close infrastructure networks is shown: 1) a traditional methodology based on a grid-cell based grouping method, and 2) a methodology based on a combined topology / Voronoi cell / density based clustering of interventions. Both methodologies exploit recent developments in the area of critical infrastructures and GISs. The differences are illustrated by using both methodologies to determine combined work programs for five spatially close infrastructure networks (electricity, gas, water, sewage, roads) in a municipality with approximately 1'500 inhabitants. The advantages and disadvantages of each are discussed.

1 INTRODUCTION

Infrastructure networks (INs), such as electricity, gas, road, sewer, and water distribution networks are some of the main assets but also some of the main cost drivers of a municipality. Therefore, interventions on these networks should be carried out in a way that minimises the costs of intervention and the disruption of the service, i.e. an optimal work program (OWP) should be determined. Traditionally, the managers of each IN produce their own work programs. Example methodologies to find OWPs for single INs can be found in Fenner, Sweeting, and Marriott (2000), Miyamoto, Kawamura, and Nakamura (2000), Stillman (2003), Cardoso et al. (2004), Arthur et al. (2009), Dehghanian et al. (2013), Zayed and Mohamed (2013), Lethanh, Adey, and Sigrist (2014). These work programs are then discussed with those responsible for the other INs in order to reduce the impact on service disruption and costs by manually combining the work programs to create synergy effects. In this paper a case study on a dynamic geographic intervention grouping method is presented. This method can be used to take into consideration the structural and functional differences, as well as the different ways that the networks are typically monitored, in an automated way. The resulting work program is compared with the work program from a static geographic intervention grouping method presented in Kielhauser, Adey, and Lethanh (2014), herein referred to as the traditional methodology.

2 METHODOLOGY

The concept of the herein presented dynamic neighbourhood methodology (DNM) is based on the concept of neighbourhood. The neighbourhood of object A is defined as the region around object A, where object B would be considered close if it lies within that region. The basic process for the DNM follows 7 steps, which are explained in the following sections and shown in Fig. 1.

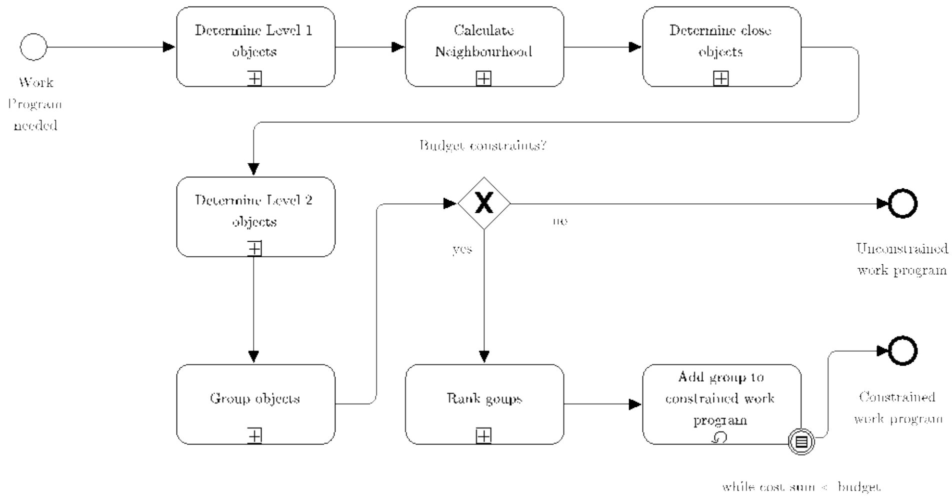


Fig. 1: Methodology Flowchart

2.1 Step 1: Determine Level 1 objects

In this process, objects with high priority (level 1 objects) are determined. In this paper, level 1 objects are defined as follows:

Level 1 objects are objects, which are in such a state (e.g. failure probability, condition, age, level of service) that an intervention on this object is justified on its own, i.e. the decision to do an intervention is independent from other objects.

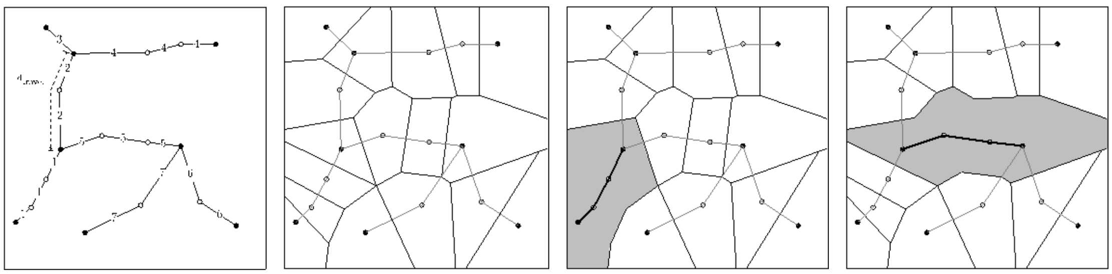
In the first step of this process, triggers for selecting an object as a level 1 object are defined. These triggers are thresholds of one value or a combination of values of the object attributes. The attribute can be of the object itself, denoted as ζ_n^I with the subscript n ($n = 1 \ldots N$) representing the object ID, and/or of the network, denoted as ξ_m^I with the subscript m ($m = 1 \ldots M$) representing the network ID. The superscript ^{I} denotes the level 1 objects. An example of the former is object condition. An example of the latter is change in network reliability. Then, objects are compared with the triggers by using a selector function f_I. It is defined as a logical function, with $f_I = 1$ (true) when an object is selected, and 0 otherwise. This selector function is applied to a set of all objects considered (based on the boundaries of the physical area to be analysed and the jurisdiction of the manager) $\vec{o}_{n,m}$ in order to obtain the logical selection vector for the level 1 objects $\vec{\delta}_{n,m}^I$:

[1] $\vec{\delta}_{n,m}^I = f_I\left(\vec{o}_{n,m}, \zeta_n^I, \xi_m^I\right)$

with $\vec{o}_{n,m}$ …object vector consisting of all objects $o_{n,m}$: $\vec{o}_{n,m} = (o_{1,1}, o_{2,1}, \ldots, o_{N,1}, o_{1,2}, \ldots, o_{N,2}, \ldots, o_{N,M})$.

2.2 Step 2: Calculate Neighbourhood

This process is used to determine the neighbourhood of all level 1 objects being investigated. This is accomplished by determining the topological neighbourhood, the distance neighbourhood and the Voronoi neighbourhood. The first two of these neighbourhoods refer to objects in the same network, the last refers to objects that are in the spatial neighbourhood of the level 1 objects but are in different INs than the level 1 object. To better explain the used three different neighbourhoods, Fig. 2a shows an example network with 7 objects. Logical nodes (denoted with ●) are nodes that signify a join between two or more objects. Geometric nodes (denoted with ○) are used to detail the object shape in between the endpoints (logical nodes). Lines with the same number are used to denote one object.



(a) Example network with objects 1-7 (b) Voronoi cells for network nodes (c) Voronoi region for network element 1 (d) Voronoi region for network element 5

Figure 2: Network definitions

2.2.1 Topological Neighbourhood

The topological neighbourhood of an object is based on the topological distance, i.e. the number of objects between two logical nodes. In that sense, two objects are neighbours if the network distance between their closest logical nodes is below a certain threshold. For example, if the focus is on object 6 and the distance threshold is 1, then objects 1, 2, 5, and 7 can be considered to be in the same neighbourhood, as at most 1 object (namely 5) has to be crossed to reach them. Mathematically, neighbouring objects can be found as follows: Starting from the incidence matrix $\mathbf{C}_m$ (i.e. a matrix describing which edges are connected to which nodes), all nodes reachable in a distance k can be calculated by:

[2] $$\mathbf{K}_m = sgn\left(\sum_{x=0}^{k}\left(\left(\mathbf{C}_m \cdot \mathbf{C}_m - diag(\mathbf{C}_m^{\prime} \cdot \vec{1})\right)^{x}\right)\right)$$

with $\mathbf{K}_m$... Reachability matrix of network m for k steps. Each element from $\mathbf{K}_m = \left[k_{m,i,j}\right]$ shows if a node is reachable from another node with a maximum of k steps. Combining those reachability matrices to a grand matrix gives the topological neighbourhood matrix $\mathbf{N}_T$:

[3] $$\mathbf{N}_T = \begin{bmatrix} \mathbf{K}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_M \end{bmatrix}$$

2.2.2 Distance Neighbourhood

As the sizes of network objects can differ by orders of magnitudes (e.g. compare one valve with a 300m stretch of straight pipe), neighbourhoods can also be defined by distance along the network. This is an alternative neighbourhood definition, which only takes into account the physical distance of objects along the network. For this, an object is defined as a neighbour, if it can be reached within a certain distance along the network. For example, objects 3 and 5 are within a distance *dtravel* of each other, as shown in

Fig. 2a, as their closest nodes are within a distance of *dtravel*. Mathematically, the shortest path between all logical nodes q of all objects has to be calculated (or loaded from the database if available).

[4] $\mathbf{D}_{m,q} = APSP_m(m,q)$

with $\mathbf{D}_{m,q}$ ...all logical node pairs distance matrix for network m, $APSP_m$ ...all node pairs shortest path algorithm for networks as described in Johnson (1977). The minimal distance between all object pairs is then the minimum of the minimal distances between each objects' logical nodes:

[5] $\mathbf{D}_m = min(dist(\mathbf{D}_{m,q}, \mathbf{D}_{m,q}) \mid q_m \in n_m)$

with $\mathbf{D}_m$ ...minimal distance matrix for all objects in network m, q_m ...logical nodes of network m, and n_m ...objects of network m. Comparing this minimal distance matrix with a distance threshold $d_{lim,m}$ for each network m gives the neighbourhood matrix $\mathbf{D}_{d,m}$:

[6] $\mathbf{D}_{d,m} := \mathbf{D}_m \le d_{lim,m}$

Combining those distance matrices into a grand matrix gives the neighbourhood matrix $\mathbf{N}_N$:

[7] $\mathbf{N}_N = \begin{bmatrix} \mathbf{D}_{d,1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_{d,M} \end{bmatrix}$

2.2.3 Voronoi Neighbourhood

Both topological and distance neighbourhood definitions have the prerequisite that all objects have to belong to the same network. To overcome this limitation, the neighbourhood can also be defined by Voronoi cells (Lejeune Dirichlet 1850). Figure 2b shows a Voronoi tessellation of the example network. Starting from a given set of core points p_i (in this case: all network nodes, both logical and geometric), Voronoi cells V_v consist of every point in space whose distance to p_k is less or equal to any other core point $p_{j \ne k}$. A Voronoi region R_i for one object is then the set of all Voronoi cells emanating from the nodes of that object (Fig. 2c,d). Combining those regions gives the set of all Voronoi regions $\vec{R}_{n,m}$ for all objects $o_{n,m}$. This can be used to define the neighbourhood as follows: objects A and B are neighbouring, if object B touches object A's Voronoi region. The neighbourhood can then be defined as:

[8] $\mathbf{N}_V = \vec{R}_{n,m} \prod \vec{o}_{n,m}$

with $\vec{R}_{n,m}$ ...Voronoi regions of objects $\vec{o}_{n,m}$, $\mathbf{N}_V$ ...neighbourhood matrix for Voronoi methodology and $\vec{R}_{n,m} \prod \vec{o}_{n,m}$ ...the dyadic geometric intersection between $\vec{R}_{n,m}$ and $\vec{o}_{n,m}$.

2.2.4 Neighbourhood combination

In the last activity of the dynamic neighbourhood calculation, all neighbourhoods are combined to a grand neighbourhood by combining all neighbourhood matrices to the dynamic neighbourhood matrix $\mathbf{N}_D$:

$$[9]\ \mathbf{N}_D = \mathbf{N}_T \vee \mathbf{N}_N \vee \mathbf{N}_V$$

with the symbol $\vee$ denoting the logical "or", which only returns a value of "1" if at least one input is "1". In words, two objects are neighbours if they are either topological, distance or Voronoi neighbours.

2.3 Step 3: Determine close objects

As next step, the so-called close objects are determined. Those objects are objects that are located in the neighbourhood which is defined by the dynamic neighbourhood matrix $\mathbf{N}_D$. This process ensures that only close objects, i.e. objects that are sufficiently near to level 1 objects are considered as level 2 objects in the next step, where the object condition is compared against a threshold likewise to the determination of level 1 objects. Mathematically, the whole process can be expressed in one equation:

$$[10]\ \vec{\delta}^{C,D}_{n,m} = ((\mathbf{N}_D \otimes \vec{\Delta}_i)^{\mathsf{T}} \cdot \vec{1}) \nrightarrow \vec{\delta}^{I}_{n,m}$$

with $\vec{\delta}^{C,D}_{n,m}$ ...binary variable vector indicating if object $o_{n,m}$ is part of the close object set. The superscript C denotes the close objects, the superscript D denotes the DNM, and the symbol $\nrightarrow$ denotes the material non-implication, which is an operator that only returns a value of "1" if the first input is "1" and the second input is "0". In this case: an object is only a close object for the DNM if the object is within the same neighbourhood as a level 1 object but has not been already selected as a level 1 object.

2.4 Step 4: Determine Level 2 objects

In this subprocess, objects with lesser priority (Level 2 objects) are determined. In this paper, level 2 objects are defined as follows.

Level 2 objects are objects, which are in such a state (e.g. failure probability, age, level of service) that an intervention on this object is not justified on its own but the synergies created by doing a combined intervention with a level 1 object justify an intervention, i.e. the decision to do an intervention is dependent on other proximate objects.

The selection process is similar to the one for level 1, just with different trigger values.

2.5 Step 5: Group objects

In this process, the level 1 and level 2 objects which have been identified in the previous steps, are combined into intervention groups, i.e. sets of interventions that are executed together, using the DBSCAN algorithm as described in (Ester et al. 1996). This algorithm takes the centroid point coordinates of the objects requiring intervention and two values as inputs: 1) Eps - the maximum distance in which two interventions should be grouped into one intervention group, and 2) MinPts - the number of interventions, that form an intervention group. Then, the points are classified as either belonging to a group or being single objects. If they belong to a group, then the interventions are included in the work program as a group, otherwise it has to be discerned between level 1 and level 2 objects. Level 1 single objects are also included in the work program as intervention, despite not being grouped, because per definition level 1 signifies that an intervention is justified even without grouping, whereas level 2 objects belonging to no group will be discarded. Mathematically:

[12] $\mathbf{I}_D = DBSCAN(\vec{\delta}^{I}_{n,m} \wedge \vec{\delta}^{II}_{n,m}, Eps, MinPts)$

with $\mathbf{I}_D \ldots$ intervention group matrix for the DBSCAN clusters. The cost of each group is the sum of the setup costs c_j for executing all interventions in group s_j and the unit costs without setup costs for each intervention on each object of each network $c_{n,m}$ multiplied by its size $u_{n,m}$.

[13] $C_j = c_j + (((\mathbf{I}_D \otimes (c_{n,m})^{\mathrm{T}}) \otimes (u_{n,m})^{\mathrm{T}}) \cdot \vec{1})$

with $C_j \ldots$ cost vector of intervention in all groups s_j. The WP is simply the whole set of intervention groups, i.e. all rows of $\mathbf{I}_D$. The total costs are then simply the sum of the cost vector C_j:

[14] $C_{WP} = \sum_{j=1}^{J} C_j$

2.6 Step 6: Rank groups

If there are constraints (e.g. budget constraints) and not all intervention groups can be included in the work program, a priority (related to the consequences that would be incurred due to service interruption if a failure occured) is calculated so that the groups with the highest priority can be included in the work program. It is in this methodology based on two components: a) The object priority value, which relates to the object itself and its role in the network, e.g. for a sewer network object how many upstream objects will also be blocked if this object has to be blocked for maintenance, and b) a multiplicative factor related to the location, e.g. population density of the area with service interruption. Although this is multiplicatively connected with the object priority value, it is kept separate because the population density for example is not network dependent, whereas the object priority value is. Mathematically:

[15] $\vec{W}_i = ((\mathbf{I}_D \otimes (\lambda_{n,m})^{\mathrm{T}}) \cdot \vec{1}) \circ \vec{\lambda}_i$

with $W_i \ldots$ priority vector of group s_i, $\vec{\lambda}_{n,m} \ldots$ object priority vector based on object n and network m, $\vec{\lambda}_i \ldots$ group priority vector based on group location, and $\circ$ representing the element-wise multiplication of vectors. From there, the rank r_i of a group can be calculated:

[16] $r_i = rank(sort(W_i | desc.))$

This results in the group with the highest priority W_i having the lowest rank r_i.

2.7 Step 7: Add group to constrained work program

The work program with constraints is constructed from the unconstrained work program: The groups included in the unconstrained work program are added one at a time to the constrained work program (i.e. a selection variable Δ_g^c is changed from 0 to 1) starting with the group with the highest priority (i.e. where $r_i = 1$, then where $r_i = 2,3,\ldots$ etc.) and then the budget is checked. If the budget is not exceeded, the group is kept and the group with the next highest priority is tried. This process is repeated until the sum of the costs of the groups in the work program reaches the budget limit C_{lim}:

[17] $$\Delta_g^c = 1 \;\; for \sum_{g_i(r_i=1)}^{g_i(r_i=N)} \left(C_i\right) \leq C_{lim} \;\; or \;\; 0 \;\; otherwise$$

with $\Delta_g^c \ldots$binary vector indicating inclusion in WP (1=yes, 0=no), $C_{\mathbf{lim}} \ldots$budget limit. The WP is the whole set of intervention groups $\mathbf{I}_D$ subsetted by the inclusion variable Δ_g^c:

[18] $$\mathbf{I}_D^c = \left(\mathbf{I}_D \otimes \Delta_g^c\right)$$

The total costs are the sum of the cost vector C_i times the inclusion vector.

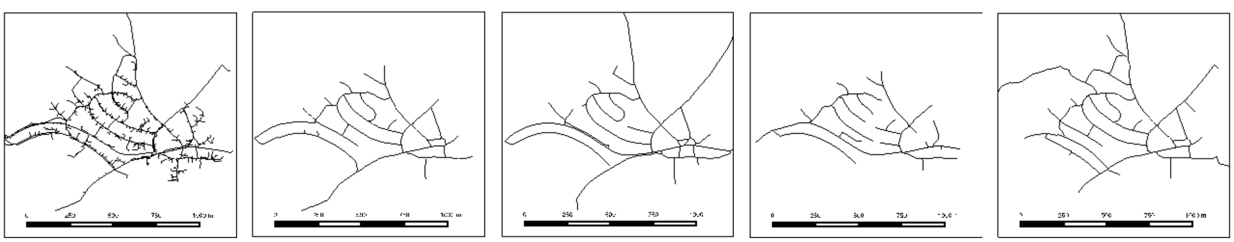
[19] $$C_{WP_{1c}} = \sum_{g=1}^{G} \left(C_i \circ \Delta_g^c\right)$$

3 CASE STUDY

The DNM was used to determine a work program for five proximate municipal infrastructure networks. This work program was then compared with the work program generated using a traditional methodology Kielhauser, Adey, and Lethanh (2014). The advantages and disadvantages of each will be discussed.

3.1 Overview

The infrastructure networks (electricity, gas, roads, sewage, and water) used in this case study belong to a municipality with a population of ca. 1'500. The single network maps are shown in Fig. 3a – Fig. 3e.



(a) Electricity (b) Gas (c) Roads (d) Sewer (e) Water

Figure 3: Network maps

The objects in the electricity, gas, and water networks were considered to be in one of 2 condition states, operational and not operational, the objects in the sewer network were considered to be in one of 5 discrete condition states, and the road objects were considered to be in a continuous-range condition state between 0 and 5 (Tbl.1). The thresholds for level 1 and level 2 are shown in Tbl. 2, the costs for the interventions (given in monetary units mu) in Tbl. 3.

Table 1: Network characteristics at the time of investigation

	Length	Objects	No.of condition states	Condition state (distribution)				
	[km]	[-]		1 (as new)	2	3	4	5 (defunct)
Electricity	20.6	2'226	2	100%	-	-	-	0%
Gas	6.8	351	2	100%	-	-	-	0%
Water	8.8	182	2	100%	-	-	-	0%
Road	9.7	104	cont.	(0-1] 2.9%	(1-2] 25.0%	(2-3] 44.2%	(3-4] 23.1%	(4-5] 4.8%
Sewer	5.3	215	5	97.2%	2.8%	0%	0%	0%

Table 2: Threshold values for levels 1 and 2

	Electricity	Gas	Roads	Sewer	Water
Type	Failure probability	Failure probability	Condition state	Transition probability	Failure probability
Level 1	0.025	0.025	3.67	0.020	0.020
Level 2	0.013	0.013	3.10	0.005	0.005

Table 3: Intervention types and costs

	Electricity	Gas	Roads	Sewer	Water	Setup cost
Type	Replacement	Replacement	Replacement	Replacement	Replacement	Cost per intervention setup
Value	100	150	350	250	300	1'500
Unit	[mu/m]	[mu/m]	[mu/m]	[mu/m]	[mu/m]	[mu]
Effect	As-new condition	As-new condition	As-new condition	As-new condition	As-new condition	

Using steps 1-7, a work program (with a budget constraint of 200'000 mu) was calculated and compared to the WP obtained by using the method described in Kielhauser, Adey, and Lethanh (2014). To facilitate the comparison, the work programs will be referred to as WPD (with the D representing the DNM) and WPT (the T representing the traditional methodology). Tbl. 4 shows the results.

Table 4: Results

	Objects	Intervention		Cost	Avg. cost per			Removed	B
		Groups	Length		Group	Object	Length	Age units	
	[-]	[-]	[m]	[mu]	[mu/1]	[mu/1]	[mu/m]	[y]	[mu/(m*y)]
WPT	4	4	861.6	287'387	71'847	71'847	334	7'965	36.1
WPD	6	4	874.5	288'672	72'168	48'112	330	8'724	33.1

This table shows that the traditional methodology selects 4 objects for the work program. A total amount of 861.6m will be renewed. With only 300'000 mu available, the total cost is 287'387 mu, which means a budget utilisation of 96%. The DMN selects 6 objects with a total length of 874.5m for the work program.

A total amount of 874.5m will be renewed. With total costs at 288'672 mu, the budget utilisation is also 96%. To be better able to compare the methodologies, we have used a proxy B which is the average amount of financial resources used to improve the age of the infrastructure, expressed as monetary units divided by the amount of removed age, multiplied by extent of the object. This indicator is based on the concept, that if it is assumed that an adequate level of service is always provided, the main goal of an infrastructure manager becomes the improvement of the infrastructure for the least cost. In this case, the best work program is the one that gives the highest improvement in the infrastructure provided for the least cost. For example, when a gas pipe is 70 years old and is replaced with a new identical pipe, it is considered to have improved the condition of the infrastructure by 70 years. When this is coupled with the costs to improve the condition, one gets the costs per time.

4 COMPARISON

One of the first things that can be seen by comparing the two methodologies is the difference between the numbers of interventions proposed. For the case study, this difference originates in a part of the network that is shown in Fig. 4. The left half shows the WPT, the right shows the WPD. The dashed line represents the boundary of a grid cell. The line weight represents the different levels (thick: level 1, medium: level 2, grey: no selection). For the WPT, it can be seen that within the grid cell, the level 1 object also triggers an intervention on the adjacent level 2 object, because it is in the same grid cell. For WPD, additionally, another object is included, as it is only at a topological distance of 1 from the level 1 object. This shows the main benefit of the DNM: as the neighbourhood is calculated dynamically from the network, such boundary situations can be taken into account in contrast to the traditional methodology.

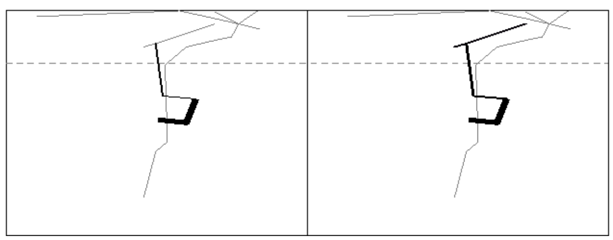


Figure 4: Difference in work program

As can be seen in Tbl. 4, the cost/improvement ratio B varies between the work programs. The work program determined using the DNM gives the best (i.e. lower) ratio (33.1 vs. 36.1). Therefore, the DNM performs better than the traditional methodology. This is due to the synergy effects created by the topological approach in contrast to the traditional approach.

5 CONCLUSION

In this paper, two methodologies to determine work programs for municipalities possessing multiple infrastructure networks were investigated and compared. In the investigated example, the methodologies were used to determine work programs for five infrastructure networks in a municipality with a population of ca. 1'500. It was found that the DNM for grouping objects in need of intervention within a network and on multiple networks leads to improvements over the traditional methodology. This demonstrated that the explicit consideration of the proximity of objects within multiple networks should be systematically done. It was found that the DNM leads to work programs that generated lower per unit improvement costs than the traditional methodology. This is because the traditional methodology relies on predefined grid cells, while the DNM dynamically calculates those from the individual network data.

Keeping this in mind, future research in this direction should include:

- the enhancement of the investigated methodologies to determine work programs using more direct consideration of the costs of service interruption
- the extension of the investigated methodology to determine work programs over multiple time periods
- the adaptation of the investigated methodology to take into consideration functional relationships between the objects within one network and within multiple networks, e.g. the effect on the amount of water flowing in a waste water network due to interventions being executed on a sewer network and a heavy rain fall occurring
- the enhancement of the investigated methodology to better take into consideration real world costs, i.e. more accurate representations of the variant and invariant costs involved with intervening on one or more networks, and on one or more objects
- the comparison of these methodologies with a real world situation to investigate the potential advantages and disadvantages of its use in practice

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